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ABSTRACT

Error analysis performed on data sets revealed that seventh grade students used a variety of rules of operation for solving the signed-number arithmetic problems. The data sets were obtained from an experimental study in which students were randomly assigned to study either of two lessons written on a computer based education system. The two lessons used different conceptual frameworks. The verbal Postman Stories and the Number line method. This paper deals with the early stages of learning in which the students applied different rules of operation. These systematic sources of variation in the data resulted in increased dimensionality. Moreover, the variety of algorithms (rules of operation) used by the students in both treatment groups supported the assertion that the two instructional methods were at least partially responsible for individual differences in information processing. (Author/DWH)

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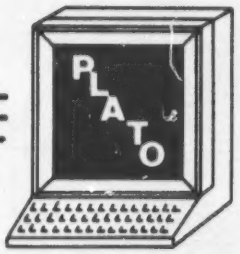

University of Illinois Urbana Illinois

EFFECT OF DIFFERENT INSTRUCTIONAL METHODS ON ERROR TYPES AND THE UNDERLYING DIMENSIONALITY OF THE TEST PART I

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Error Analyses performed on data sets revealed that students used a variety of rules of operation for solving the signed-number arithmetic problems. The data sets were obtained from an experimental study in which students were randomly assigned to study either of two lessons written on a computer-based education system. The two lessons used different conceptual frameworks; verbal stories and the number line. This paper deals with the early stages of learning

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in which the students applied different rules of operation. These systematic sources of variation in the data resulted in increased dimensionality. Moreover, the variety of algorithms (rules of operation) used by the students in both treatment groups supported the assertion that the two instructional methods were at least partially responsible for individual differences in information processing.

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The main focus of this study is on issues related to the effect of different instructional methods on the consistency and types of algorithms developed by students as well as on the underlying dimensionality of the test.

The first phase of the experiment, in which the dimensionality of an achievement test across different learning stages under two different instructional methods was investigated, was carried out in January, 1980, using about 160 seventh graders at a junior high school. The motivation for the experiment originated with the earlier studies of Tatsuoka & Birenbaum (1979, 1981). Unidimensionality of a test is essential, at present, whenever latent trait models are to be involved in order to measure an individual's location on an ability or achievement continuum underlying the test. The study demonstrated that instructional methods affect the dimensionality of test scores to a large extent. As to the stability of the responses across parallel items, the results indicate that in the early stages of learning, students tend to change their rules of operation during the test. This phenomena causes a clear violation of the local independence assumption which is one of the most essential assumptions of the Item Response theory (or latent trait theory).

The results of this study underscore the need for combining the efforts of cognitive psychology and psychometrics, as Glaser mentioned in his 1981 paper.

The authors summarize empirical results of error analysis and factorial studies based on the first phase of experimental data in this study. A further completed study will be reported in a future research report.

Methodology

The Data Collection Procedures

The dataset used in this study (to be referred hereafter as the "January data") was collected in the winter of 1980. Two methods for teaching the concept of signed numbers, one entitled: "The Postman Stories " and the other "The Number-Line Method " were implemented on the PLATO system. 157 seventh graders from a Junior High School were randomly assigned to study the topic under one of those methods without any assistance from teachers. Each instructional unit was divided into two sessions; one for teaching addition and the other for teaching subtraction. After addition was introduced and practiced (including a check-up with feedback) a test consisting of 52 free response items representing 13 tasks of 4 parallel items each was administered on the PLATO system. A copy of this test is presented in Appendix 1. Eighty-one students from both experimental groups completed that test twice. Their responses constitute the four datasets used in the current study.

The Instructional Methods

The Postman Stories. This method for teaching signed numbers is part of the Madison Mathematics Project (Davis, 1964). This method associates positive and negative integers with checks and bills, respectively. Addition is represented by a postman bringing something (a check or a bill), while subtraction corresponds to the postman's taking something from the home.

In the programmed version of this lesson which was used in the January experiment addition was presented by six types of tasks. The first one of adding two positive integers appeared on the screen both as a number story: "+2 + +5" and as a Postman Story: "On Monday, the postman brings you a check for $5 and he brings a check of $2. You are richer by $7. The other five tasks appeared on the screen either as a postman story to which the student had to fill in the number story (i.e. the equation) or vice versa.

Subtraction was presented in a simmilar way. A problem of subtracting two positive integers appeared on the screen both as a postman and a number story as follows: "+5 - +2 = +3", "On Monday the postmans brings you a check for $5 and he takes away a check for $2. You are richer by $3." For the other five tasks in subtraction the student had to fill in either the number story or the postman ones. Table 1 presents the structure of the problems that were solved during the lesson. Each problem had to be solved correctly before the student could preceed to the next one. Whenever a mistake was made the screen would flash in that part of the problem and a short explanation of the situation which was presented in the problem would appear before the student was asked to try it again.

The Number-Line method. This geometric method for teaching signed numbers introduces the concept of negative integers by means of moving a pointer to the left of zero on the number line. This unit was originally implemented on PLATO by T. Weaver. In that version of the lesson, rules were given as to which direction to move the pointer for addition and subtraction of integers. These rules were deleted from the lesson in the January experiment in order to make it comparable to the Postman lesson. The lesson begins with an introduction of the number line and the location of positive and negative integers on it. The students practice locating numbers on the number line by moving a pointer. After getting acquainted with the number line, the students are asked to solve an addition problem of two whole numbers: 5 + 2 by using the pointer. Five more addition problems, consisting of different combinations of positive and negative integers, are presented. The students are expected to generate the rules as to which direction to move the pointer in order to solve them. (The problems are identical to those presented in the "Postman" lesson. For the structure of the problems see Table 1.)

The two computerized instructional units, the Postman stories and the Number Line method, as used in the January experiment, were meant to

Table 1

The types of problems presented in the computerized version of the "Postman Stories" and the "Number-Line" instructional methods.

	addition	subtraction
1.	+L + +S	+L - +S
2.	+L + -S	+L - -S
3.	+S + -L	+S - -L
4.	-L + -S	-L - +S
5.	-S + -L	-S - +L
6.	-S + -L	-S - -L

teach the concept of signed numbers in two different representations. Both of these approaches deliberately refrained from stating rules for solving problems, in contrast to the teachers' method, which gave rise to data collected in November, 1979 (see Birenbaum & Tatsuoka, 1980). The difference between the November and the January teaching methods in this respect is simlar to the one made by Winograd (1980) concerning the pre-AI programs for language and the basic AI model. Winograd emphasizes that the critical element in the AI basic model is the explicit manipulation of a formal representation. In this model as Winograd states: "Operation carried out on the representaion structures are justified not by facts about language, but by the correspondence between the representation and the world being described. This is the sense in which such programs were said to "understand" the words and sentences they dealt with where the earlier machine translation programs had manipulated them without understanding" (ibid p.211). Both of the teaching units had identical check-up problems, one set after addition and the other after subtraction. These check-ups consisted of items parallel to the items the student practiced solving in the lesson. The correct answers were provided during the lesson and the check-ups.

The data analysis procedures. The data analysis procedures used in this study are similar to those described in Birenbaum & Tatsuoka (1980) for the November data. Figure 1 summarizes the design of the data collection and analyses procedures.

The Effect of the Point of Time in the Learning Process and the Type of Instruction on the Consistency of Responses Across Parrallel Items

Consistent responses across parallel items can be considered as an indicator of the student's level of "mastery" or confidence in his hypothesized rule. Psycholinguistics explains language acquisition by children as a process of hypothesis generating and then testing them against the language spoken around them (see for example Moskowitz, 1978). Similarly, when a student is faced with a problem solving task in mathematics, he will generate an hypothesis (e.g., rule) and test it on different problem types. The rule will be changed or modified only when the student recognizes "new circumstances" under which his hypothesis can no longer be confirmed. As stated by Davis et. al. (1979) concerning this matter: "...discriminations develop only as finely as they are needed..." (p.116). Thus, when the student is still in a stage of testing and modifying his hypothesis, i.e., when mastery, even for incorrect rules has not yet been achieved, one is likely to find less consistent responses across parallel subtests. This is due to the fact that each new task provides information which either confirms the hypothesis or disconfirms it, in which case the hypothesis is modified and tested on the next task and modified again if necessary as the next task is approached.

We'll illustrate this process with examples from interviews with two students who were taking the PLATO lessons on signed numbers.

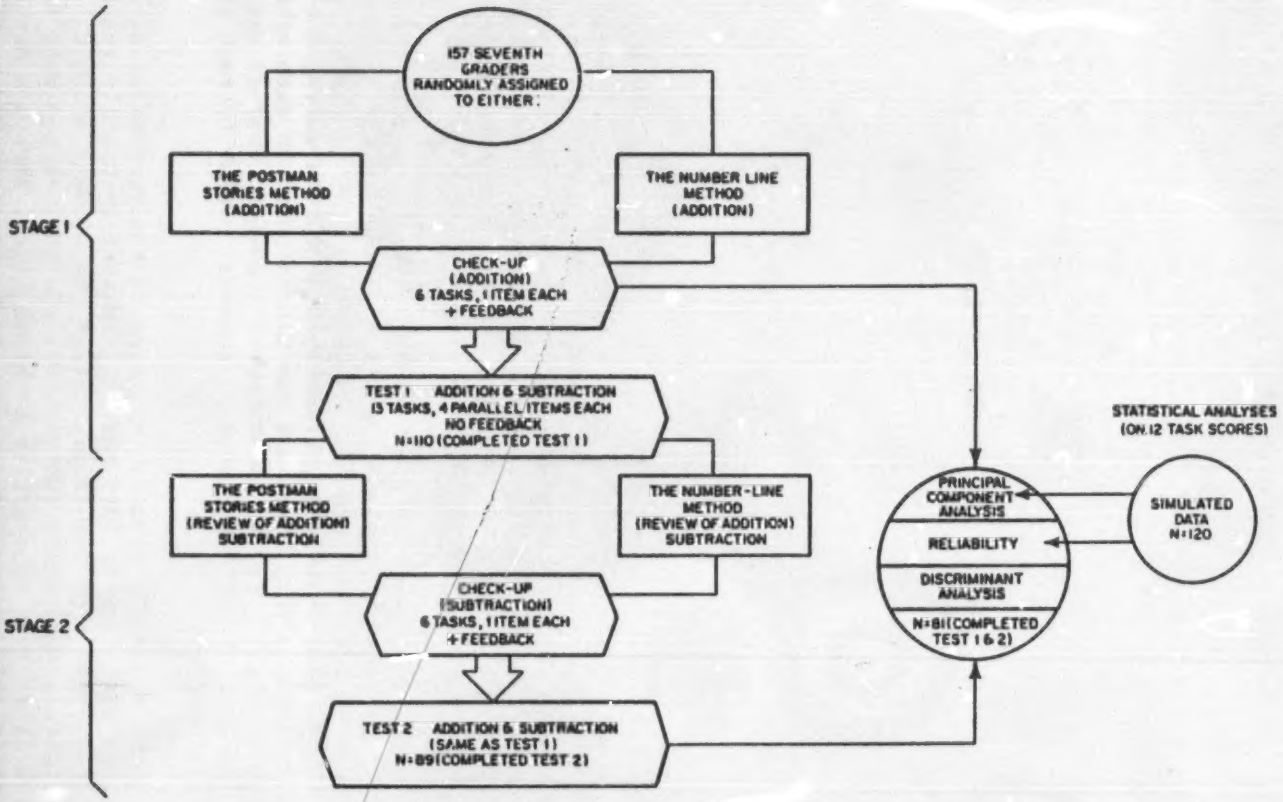
157 SEVENTH GRADERS RANDOMLY ASSIGNED TO EITHER:
THE POSTMAN STORIES METHOD (ADDITION)
THE NUMBER LINE METHOD (ADDITION)
STAGE 1
CHECK-UP (ADDITION) 6 TASKS, 1 ITEM EACH + FEEDBACK
TEST 1 ADDITION & SUBTRACTION 13 TASKS, 4 PARALLEL ITEMS EACH NO FEEDBACK N=110 (COMPLETED TEST 1)
THE POSTMAN STORIES METHOD (REVIEW OF ADDITION) SUBTRACTION
THE NUMBER-LINE METHOD (REVIEW OF ADDITION) SUBTRACTION
STAGE 2
CHECK-UP (SUBTRACTION) 6 TASKS, 1 ITEM EACH + FEEDBACK
TEST 2 ADDITION & SUBTRACTION (SAME AS TEST 1) N=89 (COMPLETED TEST 2)
STATISTICAL ANALYSES (ON 12 TASK SCORES)
PRINCIPAL COMPONENT ANALYSIS
RELIABILITY
DISCRIMINANT ANALYSIS
N=81 (COMPLETED TEST 1 & 2)
SIMULATED DATA N=120

Figure 1. The Research Design

Case 1. The response pattern presented in Table 2 for the sign-numbers test was given by a 7th grade student who studied the "Postman Stories" and was interviewed while taking the lesson and the test. During the test there was no feed-back or judgement concerning the correct answer.

The coded consistency pattern for this student is as follows. The explanation of the terminology is given in Birenbaum & Tatsuoka (1980).

Task no.	Task type	Consistency code
4	S-(-L)	0
2	-S-L	0
13	-S-+L	4
1	-S-(-L)	3
8	-L-(-S)	3
7	L-S	1
16	S-L	1
10	-L+-S	1
5	-S+L	1
15	-L+S	1
3	L+-S	1
11	S+-L	1

This response pattern indicates quite a high degree of stable answers to most of the test items. At least on three parallel subtests the student is using the same strategy for solving parallel items, (excluding tasks 4 and 2). It also indicates that the student has mastered the correct algorithm for addition of signed numbers since he consistently answers all types of addition problems correctly.

As can be inferred from this consistency pattern, the rule for subraction that this student has mastered is as follows: Always change negative integers to positive ones and add the two integers. The "motivation" behind this rule is expressed by the student during the interview[1] as follows: "When there are two minuses next to each other you change them to plus." To the interviewers question "What do you mean by 'next to each other?'" the student answers: "Next to each other is either: -[]- or []--."

Comparing the student's response-patterns on the four parallel subtests provided more insight into the process that eventually led him to generating this rule. On the first subtest the student doesn't distinguish between addition and subtraction. He applies the rule for addition to the first five subtraction problems (i.e., treats a minus operation as if it were a plus operation). It is unclear what he is doing on the 6th subtraction problem since he typed the number 11 as the answer to the problem: 13 - (-4). However, on the 7th subtraction problem, in which for the first time in the test he is faced with a

Table 2

The Complete Response Pattern to the Signed Number Test

Given by the First Student During the Interview.

	PROBLEM	st.ans.	correct		PROBLEM	st.ans	correct		PROBLEM	st.ans.	correct		PROBLEM	st.	ans.	correct
1	-6-(-8)=	-14	2	17	-1-(-10)=	11	9	33	-3-(-5)=	8	2	49	-2-1(-11)=	3		9
2	-7-9=	2	-16	18	-2-11=	9	-13	34	-4-6=	10	-10	50	-5-14=	19		-19
3	12+-3	9	9	19	7+-5=	2	2	35	15+-6	9	9	51	4+-2=	2		2
4	1-(-10)=	-9	11	20	3-(-12)=	-9	15	36	5-(-7)=	12	12	52	6-(-8)=	14		14
5	-3+12=	9	9	21	-1+10=	9	9	37	-4+13=	9	9	53	-2+11=	9		9
7	8-6	14	2	23	7-5=	2	2	39	4-2	2	2	55	9-7	2		2
8	-16-(-7)=	-23	-9	24	-12-(-10)=	22	-2	40	-11-(-2)=	13	-9	56	-7-(-5)=	12		-2
10	-14+-5=	-19	-19	26	-10+-1=	-11	-11	42	-7+-5	-12	-12	58	-10+-8=	-18		-18
11	3+-5=	-2	-2	27	2+-11	-9	-9	43	6+-8=	-2	-2	59	1+-10=	-9		-9
12	13-(-4)=	11	17	28	0-(-9)=	9	9	44	6-(-4)=	10	10	60	0-(-2)	2		2
13	-3-+12=	15	-15	29	-2-+11=	13	-13	45	-7-+9=	16	-16	61	-4-+6=	10		-10
15	-6+4=	-2	-2	31	-5+3=	-2	-2	47	-4+2=	-2	-2	63	-8+6	-2		-2
16	2-11	13	-9	32	5-14=	-9	-9	48	7-16=	-9	-9	64	4-13=	-9		-9

subtraction problem that has an explicit plus sign for one of the integers, i.e., -3 - +12, he suddenly realizes that something is wrong with his former rule. Thus, from here on he switches to his new rule concerning the two minuses being close to each other. He extends his rule to the case of three minuses (problem 17) which according to his conception has two cases of two minuses close to each other. In his responses on the second subtest it can be noticed that the "new" and the "old" rules are still competing. Thus, the problems in tasks 2 and 4 are still answered using the "old" rule (see responses to items 18 and 20). (This causes the two codes of zero in the consistency pattern for tasks 2 and 4). However, on the next two subtest the "new" rule stabilizes and is being used throughout the rest of the subtraction problems (except for tasks 7 and 16 which are treated as a different category, as stated by the student in the interview: "'cause it is a regular case of two positive numbers."

Case 2. The second example of hypothesizing a rule, testing it and switching to another rule when the first one breaks down is illustrated below using data from another interview. This student was interviewed while she was studying the "Number-Line" lesson and the part of the protocol we are referring to deals with the practice problems that were given at the end of the subtraction unit. At this point feedback regarding the correct answers was provided.

The first inductive rule which this student generates is as follows: "on the right side of the number line you go to the right to subtract and to the left to add. On the left side of the number line you go left to subtract and right to add"[2]. This incorrect rule yields correct answers to problems such as: $4 - (-3) = 7$; $1 - (-3) = 4$; $-2 - 5 = -7$; $-4 - 2 = -6$ and the student gets more and more self assured about her rule as can be noticed in her remarks: "Ohh, I'm right; I was right!!" (Ibid p. 10).

However, this rule breaks down with problems involving two numbers with the same signs such as: $-4 - (-3)$. Following her rule the student gets an answer of -7. She seems to be surprised to see the message that her answer is wrong and after practicing some more problems she induces the following new rule: if there is an explicit plus sign in the problem, add and put the sign of the first number. Or, as stated by the student when explaining the way she solves the problem $-1 - (+2)$: "I added because it says one plus two. Now I get it! One minus plus two. O.K., I don't know why these things are here. It says one and then in the parentheses it says plus two, and this sign right here [the sign in front of the first number] you are supposed to put on the answer." As can be seen from this example and form the previous one, in the stage of learning, when mastery has not yet been reached, students are hypothesizing their algorithms and testing them. The change or modification of an algorithm occurs only when the previous rule seems to break down. Thus, any kind of feedback, either of the form of judging the answer and providing the correct one or even the next type of task in the test serves as a reference against which the student is testing his hypothesized algorithm.

It should be noticed that if indeed a test serves sometimes as a learning experience causing students to switch strategies on the basis of clues they got from the different tasks or types of questions, then applying latent trait models to such tests is inappropriate. It is a clear violation of the local independence assumption which is one of the most critical assumptions of ICC theory (see Lord & Novick 1968). The following is one of the equations given by Lord & Novick (1968) as a definition of "local independence":

$h_1(y_{1*}|\ \theta;\ y_{2*},y_{3*},\ldots,y_{n*}) = f_1(y_{1*}|\ \theta)$ (eq.16.3.2 p.361)

This shows that under local independence, the conditional distribution h_1 of an item score y_{1*} for fixed values of θ, $y_{2*},y_{3*},\ldots y_{n*}$ doesn't depend on other item scores $y_{2*},y_{3*},\ldots y_{n}$. Thus, for respondents with the same θ value, any association between the items should disappear.

In pointing out different kinds of arguments to defend the introduction of the local independence requirement, Mokken (1971) states: "In the first place there are some general substantive considerations concerning proper measurement which may give rise to this principle. What these considerations amount to is that when we have determined the value of a subject and kept it constant, all systematic variation in the outcomes of measurement of that value should disappear, leaving only purely random variation" (ibid p.76). This of course is assuming that we are measuring a stable trait. However, as was shown in section of this report, when the test is given during the stage of learning, when the algorithms haven't yet been stabilized, but are rather being modified during the test, we can't talk of a stable latent trait. At most we consider a latent state which is likely to change during the testing situation. Thus, the local independence assumption which implies that the order of the test items should not affect the estimates of the examinees' ability level is clearly violated in the case where the order of the test items does make a difference. In such a case there exists a systematic variation which is attributed to another related dimension - another latent state which reflects another generated algorithm. Therefore, any measurment model, including the ICC family, that assumes a static rather than a dynamic trend in the responses on a given test would be inappropriate for use in those stages in the instructional process in which the algorithms are not yet stabilized.

A Comparison of the Consistency of Responses in the Two Experimental Groups

In order to compare the consistency of responses across parallel items in the two experimental groups, a discriminant analysis was performed. A score of 1 was assigned whenever at least 3 out of 4 parallel items got the same code (1, 2, 3, or 4) otherwise a score of zero was assigned.

The results of this anlysis for the first test (the one that was given at the end of the addition lesson) are presented in Table 3. As can be seen in the table, there is an overall significant difference between the two experimental groups with respect to the consistency of

Table 3

Means, Canonical Correlation and Standardized Canonical Discriminant Function Coefficients of Consistency for the Two Treatment Groups in the First Test

	group	N	Means
"Postman"	1	39	-.49
"Number Line"	2	42	.45

λ	18
Rc	.43
$(1-\Lambda)100$	18.40
p<	.01

Standardized Function Coefficients

Task No.	Task Type	
2	-S-L	.61
13	-S-+L	-.46
7	L-S	.54
16	S-L	-.68
10	-L+-S	-.37

responses. The dimension along which the two groups differ involves four of the subtraction tasks and one addition task. The group with the higher Centroid (group 1) is characterized as having lower consistency scores on tasks: 13) -S-+L; 10) -L+-S; 16) S-L and higher consistency scores on tasks 2) -S-L and 7) L-S. Appendix 2 presents the percent of responses for every task on each of the five coded categories (0 - 4) where 0 indicates inconsistent responses across parallel items and codes 1 - 4 indicate that at least 3 out of 4 responses on parallel items fell into this coded category. As can be seen in this table, 41% of the students in group 2 gave inconsistent responses on task 13 compared to 21% in group 1. The most common error in the latter group is the one coded as E2. 54% of the students in group 1 consistently responded in this category as compared to 33% in group 2. In task 10, 43% of the students in group 2 gave inconsistent responses compared to 28% in group 1. 46% of the students in group 1 answered the items in this task correctly, as compared to 26% in group 2. In task 16 the rate of inconsistent responses in group 1 was 18% whereas the inconsistency rate for the other group was twice as much (36%). In group 1, 56% of the students consistently answered task 16 items correctly as compared to 45% in group 2.

In task 2, however, 41% of the students in group 1 gave inconsistent responses compared to 24% in group 2. Only 10% in group 1 and 21% in group 2 consistently answered correctly items in this task. The most common consistent error was E3. In task 7 21% of the students in group 1 as compared to 12% in group 2 gave inconsistent reponses. 56% in group 1 and 45% in group 2 is the rate of correct answers in this task. The most common error was E4: 38%(!) of the students in group 2 and 18% of the students in group 1 consistently committed this error (e.g., getting an answer of -2 to the problem: 8-6).

Since the two experimental groups were exposed to exactly the same tasks throughout their entire learning experience, the significant differences in the consistency rate between the two groups seems to imply that different information processings are involved. In the second test however, the overall difference in consistency was not significant ($p<.07$). Table 4 presents the consistency rates in each of the two tests for each of the experimental groups. As can be seen in the table, the highest consistency rate was given in the first test by group 1. At least 50% of the tasks in that test were systematically answered by 84% of the students in group 1 compared to 65% of the students in group 2. It should be noticed that this rate of consistency is far lower than the one found in the November data. In that case, the test was given after a three-week period of intensive classroom instruction including practice exercises, quizzes and tests on this topic. At that point the students already had pretty stable or fixed algorithms and that was reflected in the high rate of consistent responses in the test. 92% of the students gave systematic answers on 13 - 16 test tasks (excluding those students who consistently answered correctly). These figures clearly indicate that the stage in the instructional process, in which the test is given, does make a

Table 4

Consistency Rate: Percentage of Consistent Responses to 12 Test Tasks.

No. of Tasks consistently answered		Postwar		Number Line	
		T1	T2	T1	T2
n	(12)	15	11	11	12
n-1	(11)	7	17	10	14
n-2	(10)	22	8	15	7
n-3	(9)	17	13	16	16
n-4	(8)	14	15	5	11
n-5	(7)	9	11	8	7
n-6	(6)	5	7	14	14
n-7	(5)	5	--	10	3
n-8	(4)	1	8	3	2
n-9	(3)	--	2	3	7
n-10	(2)	--	4	3	2
n-11	(1)	5	4	2	5

difference, as far as the consistency of the responses across parallel item is concerned.

Are The Algorithms Developed by Students, Following Two Different Instructional Methods, The Same?

The two teaching methods used in the January experiment differ from each other with respect to the type of representation they offer for teaching the concept of signed numbers. The "Number-Line" method uses a representation from the geometric domain whereas the "Postman Stories" method relies on its representation of a common bookkeeping experience. Both methods as presented in the January experiment deliberately refrained from stating explicit rules for solving the problems. Students were expected to develop their own algorithms based on the representation. Thus, it becomes interesting to compare the algorithms constructed following each of these instructional methods. The following is a complete list of the algorithms identified from the students' response patterns in the two experimental groups. The algorithms are divided into three categories: those that were common to the two instructional methods and those that were unique to each method. The algorithms for subtraction and addition are described separately since the majority of the students distinguished between addition and subtraction operations and used different rules for each operation. (Appendix 3 presents the response-patterns and the number of students whose answers match those patterns.)

A) Algorithms identified in common response-patterns for two instructional styles.

Subtraction: 1st test (T1)

Please note that the numbers refer to the response patterns presented in Appendix 3. N = Number line, and P = Postman stories.

N1,P1)[3] Subtracts |L| - |S|. Puts the sign of the number with the larger absolute value in the result.
N4,P5) Subtracts |L| - |S|. Puts the sign which is attached to the second number in the result.
N1,P1) Subtracts |L| - |S|. Puts a minus sign in the result.
N5,P7) Uses the rule for addition after changing the operation sign from a minus to a plus. (Except for tasks 7 and 16).
N6,P9) Same as N5,P7.
N2,P4) When there is an explicit plus sign in the problem adds and puts a plus sign in the result. When the only explicit signs are minuses, subtracts and puts a minus sign in the result.

Subtraction: 2nd test (T2)
N7,P7) Adds the absolute values. Puts a minus sign in the result if at least one number has a minus sign. Otherwise puts a plus sign.

Subtraction: T1, T2
N11(T2);P13(T1): Subtracts |L| - |S|. Puts a plus sign in the result.
Addition: T1
N(C);P(B): When a minus sign is attached to the second number, subtracts |L| - |S|. Otherwise, adds the two absolute values. Puts a plus sign in the result.

Addition: T2
N(C);P(D): Subtracts |L| - |S|. Puts the sign of the number with the larger absolute value in the result.
N(A);P(G): Subtracts |L| - |S|. Puts a plus sign in the result.

Addition: T1, T2
N(D,T2);P(A,T1): Subtracts |L| - |S|. Puts a minus sign in the result.

B) Algorithms identified in unique response-patterns for the number line method

Subtraction: T1
7) If the first number is to the left of the second one on the number line, counts to the right the number of steps indicated by the second number.
If the first number is to the right of the second one, counts to the left, the number of steps indicated by the second number.
11) The sign of the first number determines which direction to jump on the number line. When the first number is negative, jumps to the right on the number line, the number of steps indicated by the second number.
8) The operation sign detemines the sign of the result. The sign of the second number determines the operation to be taken.
3) Subtracts |L| - |S|. Puts the sign which is attached to the number with the largest absolute value in the result.
12) Adds the absolute values. Puts a minus sign in the result (except for task 7 in which subtracts |L| - |S|).
13) Subtracts |L| - |S|. Puts a minus sign the the result (except for task 7 and 16).
14) Subtracts |L| - |S|. Puts a plus sign in the result (except for task 7 for which puts a + sign).

Subtraction T2:
2) Finds the distance between the two numbers on the number-line. Puts a minus sign in the result (except for task 7).
4) Jumps to the left of the first number on the number line the number of steps indicated by the second number.
12) When the first number is smaller in absolute value than the second, adds and puts a plus sign in the result. When

the first number is the larger in absolute value, subtracts and puts a minus sign in the result.
10) When the two numbers are positive or when there is at least one explicit plus sign in the problem, subtracts and puts a plus sign in the result. Otherwise, adds and puts a minus sign.
13) The sign attached to the number with the larger absolute value determines the operation to be taken. Puts a plus sign in the result (except for task 7).

Addition: T1
B) The sign of the first number determines the operation to be taken.
E) Subtracts |L| - |S|. Puts the sign which is in front of the second number in the result.
D) Adds the absolute values and puts the sign of the first number in the result.

Addition: T2
B) When the first number is smaller in absolute value than the second, adds and puts a plus sign in the result. When the first number is larger in absolute value, subtracts and puts a minus sign in the result.
F) Jumps to the right of the first number on the number line.

C) Algorithms Identified in Unique Response-Patterns for the Postman Stories Method.

Subtraction: T1
3) Applies the addition algorithm to the subtraction problems. I.e., when the two number signs are the same, adds the absolute values. When the are different, subtracts |L| - |S|. Puts the sign of the number with the larger absolute value in the result.
10) Subtracts |L| - |S|. When the first number is the larger in absolute value, puts a plus sign in the result. When the first number is the smaller in absolute value, puts a minus sign.
12) When the first number is the larger in absolute value, subtracts |L| - |S|. When the first number is the smaller in absolute value, adds. Puts the sign of the number with the larger aboslute value in the result.

Subtraction: T2
1) When the first number is positive, adds and puts a plus sign in the result. When the first number is negative, subtracts and puts a minus sign in the result.
6) Subtracts |L| - |S|. Puts the sign which is attached to the number with the larger aboslute value in the result (except for task 16).

Addition: T1
D) When there is an explicit plus sign attached to the second number adds and puts a plus sign in the result. Otherwise subtracts and puts a minus sign.

H) Adds the two absolute values. When there is at least one negative number, puts a minus sign in the result. Otherwise, puts a plus sign.
F) Adds the two absolute values. Puts a minus signs in the result.
I) Subtracts |L| - |S|. Puts a plus sign in the result.
G) Subtracts |L| - |S|. Puts a minus sign in the result.

Addition: T2
A) When the first number is positive, adds and puts a plus sign in the result. When the first number is negative, subtracts and puts a minus sign in the result.
F) When the first number is the smaller in absolute value adds. Otherwise subtracts |L| - |S|. Puts the sign of the first number in the result.
B) Adds the two absolute values. Puts the sign of the first number in the result (except for task 16).
C) Adds the two absolute values. Puts a plus sign in the result.
I) When the first number has no explicit number sign attached to it, adds the two absolute values and puts a plus sign. Otherwise answers correctly.

The response patterns that were common to both experimental groups seem to bear no clear relation to either one of the representations that were used during the instructional process. The algorithms identified from these responses can be explained as reflecting a mental process that refers to the syntactic attributes of the problem rather than to the semantic ones. Factors such as the sign of the first number, the location of the number with the larger absolute value, etc. seem to play an important role in determining whether to add or subtract the numbers and which sign to choose for the result. Some responses reflect no distinction between subtraction and addition operations. Others indicate a lack of distinction between integers with different signs and integers with the same signs.

Among the algorithms that where identified from unique patterns in the number-line data, there are some that are closely related to the representation used throughtout the instruction. Algorithms such as 7, 11(T1); 2, 4; F(T2) clearly indicate the use of a number line for deriving the answer. As to the unique algorithms for the postman stories, there was no clear indication that the representation was used.

It is interesting to note that even though the total test score was not significantly different between the two experimental groups the number of different algorithms for the subtraction problems in the group who studied the Number-Line method was significantly larger than that of the group which studied the Postman Stories. This again points to the important gain in information provided by error analysis as compared to the conventional scoring system.

The Postman-Stories unit presented a clear correspondence between the elements of the problems and their representation, i.e., bring/take away for operations and checks/bills for number signs. The Number-Line method didn't make a clear distinction between the operation signs and the number signs in its representation. The fact that students tended to use more the representation of the number line may perhaps be explained on the basis of the domain associations. The Number-Line method in its spatial approach uses a representation from geometry, whereas the postman approach is heavily verbal, and comes from a farther domain. It concentrates more on the concept and doesn't offer a break-down of technical steps for solving the problems, as does the other method. Such an approach was not very common in the students' past experience of studying math. In our conventional schools the boundaries between "math" and "stories" are pretty firm. It seems that students have internalized this arbitrary distinction -- they allocate separate "boxes" in their minds for these "different" domains, thus making it difficult for the knowledge and experience which is built into these boxes to interact flexibly.

Students who were studying the Postman Stories admitted that they weren't using the representation presented in the lesson for solving the test problems. However, when asked to resolve the problems using the bills and checks, they got correct answers on those items which they had previously answered incorrectly.

The following are responses given in two interviews with students who studied the Postman Stories. They are similar to to the responses we got from students during the experiment and are brought here in quite a detail in order to illustrate the point we were making about the representaion of the postman stories.

Case 1.[3] The following dialogue took place after the student completed the test.

I: Now, when you were doing the problems did you ever think about bills and checks at all or did you pretty much just...?
S: I just did it.
I: Is there any reason that you can think of why you didn't think of the bills and check?
S: I don't know, I just thought that I had to do it, because there wasn't time enough I guess, I don't know...
I: OK. Let's try something like this, -4-+5, using the postman story.
S: Um, let's see, they're giving you a bill for $4.00, but then they're taking away $5.00 of a check.
I: OK, now, when you were doing this before, how were you doing this?
S: I was thinkin' that you had to change them if they were next to each other.
I: OK, so the answer you would have gotten sould be +9 I guess, right?
S: Yeah.

I: Now, getting a bill for $4.00 makes you what? What happens?
S: You lose $4.00.
I: OK, and now what does this piece here, the -+5, mean?
S: That means that he's taking away a check for $5.00.
I: And what happens? Richer or poorer?
S: Poorer.
I: OK, so what happens all together?
S: Oh, you're getting poorer all together, ok, so you're losing $9.00.
I: OK, so the answer is...
S: Negative nine.
I: Why are you losing $9.00?
S: Because first he gives you a bill for $4.00 and then he takes away $5.00 so you add them altogether.
I: OK, and so what's the final answer?
S: Negative nine.

Case 2.[4] This dialogue also took place after the student completed the test and the interviewer asked her to solve a problem from a task she failed to answer correctly before. This time he insisted she use the postman story for solving the problem.

I: How about this problem: -2-4?
S: Well, minus two and take away four and that would give you plus two.
I: Would you do that for me using the postman story?
S: Well, you get a bill for two dollars, and you get a check for four dollars.
I: What does "get" mean?
S: Brings you, you add.
I: OK, so write down what you just said.
S: -2
4 , you take away the four
I: Write it horizontally
S: -2+4=
I: Is this the same as -2-4?
S: No, he takes away, he takes away a check for $4.00. So the answer is minus six.

How are the Algorithms Reflected in the Dimensionality of the Test?

The dimensionality of achievement test data has lately become a major concern of psychometricians due to the fact that for all practical purposes, latent trait models are applicable only to unidimensional sets of data. When such models are applied to multifactorial sets of data the parameter estimates are inaccurate if obtainable at all. (Often they are not obtainable due to lack of convergence.) For example, Reckase (1979) showed that in data sets containing more than one independent factor, the three-parameter logistic model picks one factor and discriminates among ability levels on it, while ignoring the other factors. On the other hand, the one-parameter logistic model seems to estimate the sum of the factors. In data sets which have first factors

that are large relative to the other factors in the test, both methods measure the first factor.

Studies on the dimensionality of achievement data in different topics have indicated that there are always more than one major factor underlying any test data in the achievement domain (see for example, Tatsuoka & Birenbaum, 1979; Gialluca, 1980). Moreover, it has been shown that the dimensionality of a given test changes depending on the time point in the instruction when it is given. For example, Kingsbury & Weiss (1979) demonstrated, using factor analytic techniques, that data sets obtained prior to instruction and at the peak of instruction yield different factorial structures. The variance accounted for by the first factor in their study was considerably less at the pretest than it was at the peak of instruction.

It seems that a plausible explanation for the multidimensionality in achievement data, especially of the problems solving type, can be provided by the algorithmic approach. The fact that students are using various rules of operation for solving the test problems adds systematic sources of variation in the data resulting in an increase in the underlying factorial structure of the test. As for the change in the dimensionality between the pre- and the post-test -- this can be explained as being a result of the fact that in the post-test situation, students are generating their rules of operation based on a common training or coaching treatment (i.e., instruction), whereas in the pretest the influence of this "common denominator" doesn't exist. Thus, the variety of algorithms used in the pretest is larger, resulting in a larger dimensional space. (Needless to say, in such situations where the dimensionality of the test data changes between pre and post administrations, it becomes meaningless to compute gain scores.)

In order to test the effect of the algorithms on the dimensionality of the test, we generated five datasets that varied with respect to the number of algorithms underlying the responses on a set of 12 tasks. Each dataset consisted of 120 cases. A quarter of the cases were assigned random responses. Another quarter were assigned correct responses on all 12 items. For half of the cases in the first dataset, responses were generated to follow one algorithm. The number of algorithms was increased by one for each consecutive dataset, thus ranging from 1 - 5 whereas the rest of the responses were kept unchanged across the five datasets. This simulation was meant to describe a situation in which 25% of the subjects know nothing about the topic that is being tested, and thus randomly guess at the answer. Another quarter of the subjects mastered the topic perfectly and got correct answers on all the items. Half of the subjects "mastered" incorrect rules, the number of which was incremented from one dataset to the next. This simulation was for the purpose of examining the effect of the increment in the number of algorithms on the dimensionality of the test. In order to make the effect of the algorithms clearer, we simulated a rather hypothetical situation in which 75% of the responses were perfectly reliable, i.e., uncontaminated by errors of measurement.

Table 5 presents the results of a principal components analysis for each of the five generated data sets. As can be seen in the table, the magnitude of the first eigenvalue decreases and those of the second and the third eigenvalues increase as the number of algorithms in the dataset increases.

Figure 2 presents the scree test for the five datasets. The percentages of the common variance explained by the first factor range from 52.1% in the dataset that includes one wrong algorithm to 35.7% in the dataset that includes five wrong algorithms. Figure 3 illustrates the decrease in the percent of variance explained by the first eigenvalues across the five datasets. The reliability as estimated by coefficient alpha ranges from .877 in the first dataset to .804 in the fifth one. Figure 4 illustrates the decrease in the reliability across the five datasets.

We may infer from this simulation that the increase in the number of wrong algorithms used by students in solving the test problems causes an increase in the underlying dimensionality of the test.

In order to test the closeness of the simulation study results to results from real data, a principal components analysis was done on the four datasets that were collected in the January experiment. Twelve subscores, each consisting of responses to four parallel items were computed for the four datasets. Table 6 and Figures 5, 6 and 7 present the results of the principal component analysis on the four datasets. As can be seen in the table and in the figures, more than one factor constitutes the underlying common space of the test in each of the four datasets.

The first eigenvalue in both tests of the "Postman" group explains a larger proportion of common variance than do the respective eigenvalues in the "Number Line" data sets. The variety of algorithms used in the "Postman" group was smaller than that used in the "Number Line" group. This further supports our previous explanation concerning the effect of the increase in the number of algorithms on the dimensionality of the test. (See also Birenbaum & Tatsuoka, 1980.) Moreover, in both groups, the first eigenvalue in the second test explains the larger proportion of common variance than does the first eigenvalue of the first test. The first test was given at the conclusion of the addition unit, before subtraction was introduced, whereas the second test was given at the conclusion of the subtraction unit. Thus, the "common denominator" of the instruction, which has the effect of reducing the variety of algorithms, is missing from the responses to the first test. As a consequence, the first test has a larger variety of algorithms, and this results in a smaller amount of common variance being explained by the first eigenvalue. As can be seen in Table 6 and in Figure 7 the reliability coefficients (α's) for the four tests are also in the same line with the results discussed above; i.e., higher reliabilities are found for the "Postman" compared to the respective tests in the "Number Line" group, and higher reliabilities accrue to the second test compared to the first in both groups.

Table 5

Eigenvalues and percent of variance explained and reliability coefficients for the five generated datasets

dataset / Factor	1		2		3		4		5	
	λ	variance	λ	variance	λ	variance	λ	variance	λ	variavce
1	6.255	52.1	5.369	44.7	4.980	41.5	4.621	38.5	4.279	35.7
2	2.048	17.1	2.278	19.0	2.642	22.0	2.723	22.7	2.748	22.9
3	.929	7.7	1.409	11.7	1.567	13.1	1.476	12.3	1.393	11.6
4	.791	6.6	.859	7.2	.773	6.4	.794	6.6	.981	8.2
5	.544	4.5	.524	4.4	.508	4.2	.669	5.6	.74[illegible]	6.2
6	.382	3.2	.457	3.8	.413	3.4	.473	3.9	.476	4.0
7	.337	2.8	.315	2.6	.316	2.6	.344	2.9	.389	3.2
8	.210	1.8	.283	2.4	.284	2.4	.279	2.3	.333	2.8
9	.173	1.4	.174	1.4	.174	1.5	.272	2.3	.281	2.3
10	.141	1.2	.148	1.2	.141	1.2	.142	1.2	.156	1.3
11	.105	.9	.104	.9	.122	1.0	.126	1.1	.126	1.0
12	.085	.7	.080	.7	.078	.7	.082	.7	.087	.7
α	.877		.856		.839		.818		.804	

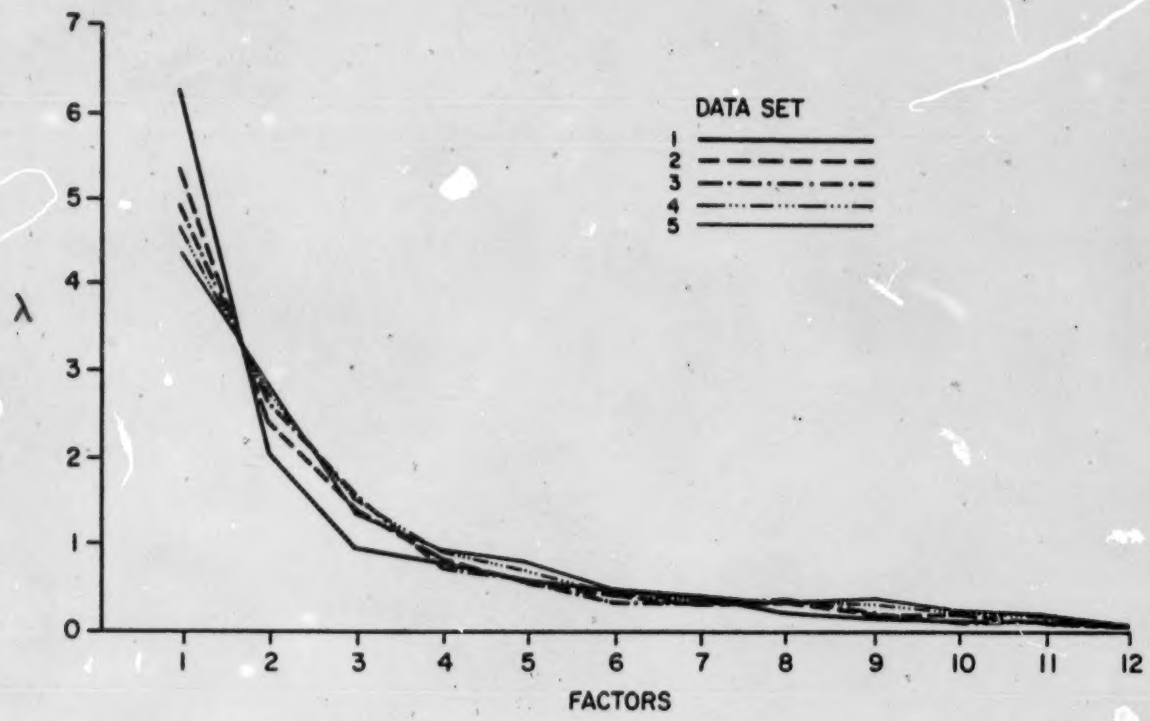

Figure 2. SCREETEST– Eigenvalues extracted by a principal component analysis for the five simulated data sets.

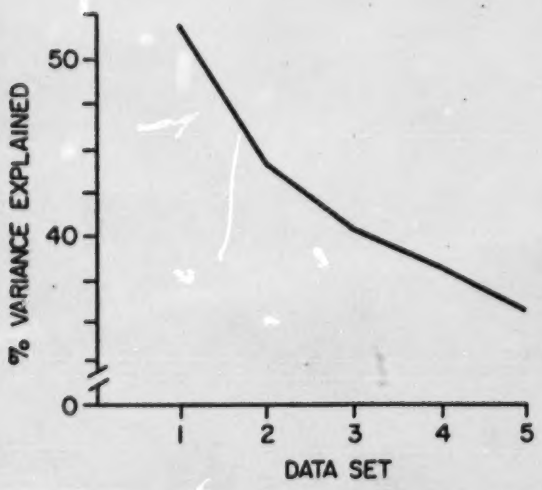

Figure 3. Percent of variance explained by the first eigenvalue across the five data sets.

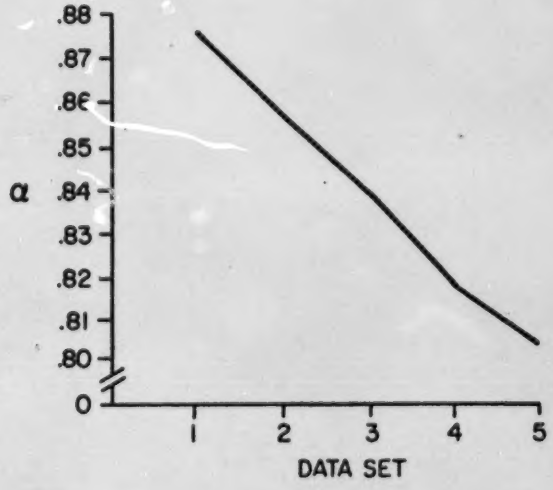

Figure 4. Reliability coefficients for the five data sets.

Table 6

Eigenvalues and percent of variance explained and reliability coefficients for the four datasets collected in the January experiment

data set / Factor	G1T1		G1T2		G2T1		G2T2	
	λ	variance	λ	variance	λ	variance	λ	variance
1	3.914	32.6	4.993	41.6	3.435	28.6	3.740	31.2
2	1.948	16.2	1.601	13.3	2.074	17.3	1.67	14.0
3	1.474	12.3	1.352	11.3	1.501	12.5	1.255	10.5
4	1.077	9.0	1.023	8.5	1.471	12.3	1.170	9.7
5	.748	6.2	.740	6.2	.781	6.5	1.072	8.9
6	.639	5.3	.600	5.0	.591	4.9	.812	6.8
7	.576	4.8	.495	4.1	.530	4.4	.582	4.8
8	.489	4.1	.360	3.0	.507	4.2	.477	4.0
9	.359	3.0	.291	2.4	.405	3.4	.421	3.5
10	.302	2.5	.198	1.6	.254	2.1	.369	3.1
11	.257	2.1	.192	1.6	.244	2.0	.246	2.1
12	.217	1.8	.157	1.3	2.06	1.7	.181	1.5
α	712		.857		.631		.764	

G1= Postman Stories.
G2= Number Line.

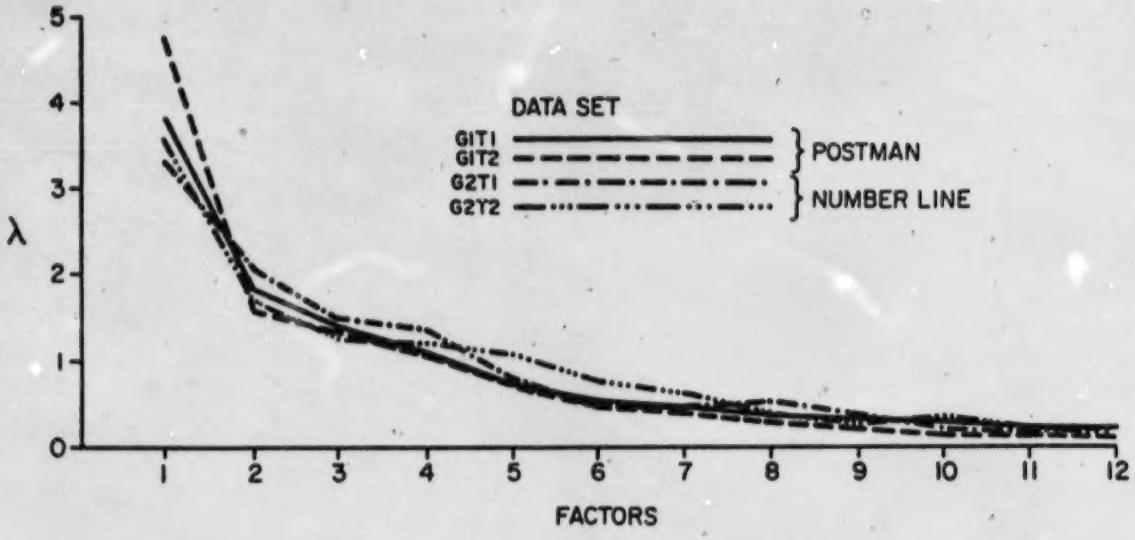

Figure 5. SCREETEST- Eigenvalues extracted by a principal component analysis for the five simulated data sets.

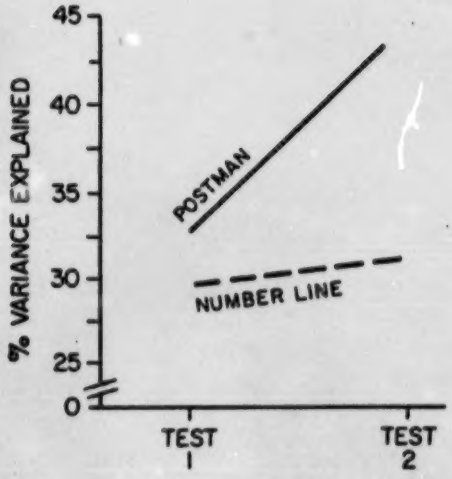

Figure 6. Percent of variance explained by the first eigenvalue in the two tests for each of the experimental groups.

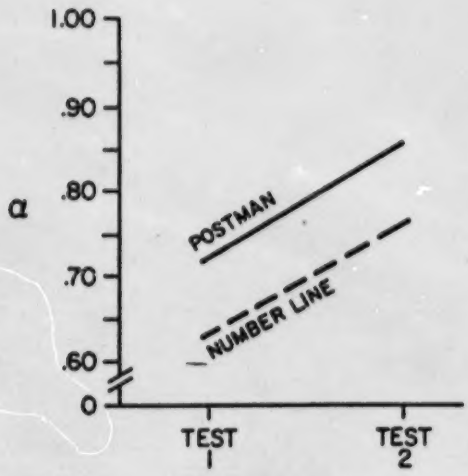

Figure 7. Reliability coefficients for the two tests in each of the two experimental groups.

Summary and Conclusions

The main focus of this study was on issues related to the effect of different instructional methods on the consistency and types of algorithms developed by students, as well as on the underlying dimensionality of the test.

The data used was collected in an experimental setting in which two instructional methods-- the Postman-Stories and the Number-Line methods-- were used for teaching signed number addition and subtraction. 157 seventh graders who had not studied the topic before were randomly assigned to each of the experimental groups. A 52 item free response test consisting of 13 tasks of four parallel items each was administered twice during the experiment. The first time -- at the conclusion of the unit on addition, before subtraction was introduced and the second time, at the conclusion of the the unit on subtraction. Eighty-one students from both experimental groups completed the test twice. Their responses on 12 of the test's tasks (those tasks which were identical to the ones used in the previous study) constitute the four datasets used in the current study.

The two different instructional methods yield an overall significant difference in the rate of consistent responses in the first test. Since the two experimental groups were exposed to exactly the same tasks throughout their entire learning experience, the significant differences in the consistency rate between the two groups seems to imply differences in the information processing mechanisms involved. This was especially noticeable in the first test, perhaps due to the fact that the test was given before subtraction was introduced. Thus, the common denominator of the instruction has not yet been affecting the students' responses greatly. This is further supported in the results of the analysis concerning the underlying structure of the test in the four datasets. The dimensionality of the first test in this analysis was found to be larger than that of the second one for both experimental groups.

The examination of the types of algorithms developed by students under the two different instructional methods resulted in identifying two categories of algorithms. One category referred to the syntactic aspect of the problem, i.e., the structure of the number sentence. The other category referred to the semantic aspect -- the conceptual representation which that sentence reflected. The algorithms were classified into three classes; those which were common to both groups and those which were unique to each experimental group. All the algorithms that were common for both groups and those that were unique to the Postman group fell in the first category. Semantic algorithms were identified only in the Number Line group. Interviews with students who were studying the Postman Stories confirmed that they were not using the representation for solving the test problems. However, when they were explicitly asked to use the representation after the test was over,

they successfully answered those test problems which they failed to answer correctly before.

The variety of algorithms used by students for solving the test problem was larger in the Number-Line group than it was in the Postman group. This finding was further confirmed in a principal components analysis that resulted in larger proportions of common variance explained by the first factor of the Postman Stories compared to the Number Line in both tests. This further supports the assertion regarding differences in the information processing mechanisms developed by students following the different teaching methods.

A comparison of the consistency rate between the January and the November data (Birenbaum & Tatsuoka, 1980) showed a large discrepency between the two. The average consistency rate in the November data was much higher than that in the January datasets. As recalled, the November data was collected in the advanced stages of the learning process after students were given sufficient drill and practice quizzes and tests. In contrast, in the January experiment many students were still in the earlier stages of learning when the experiment was over. These results indicate that the point of time in the learning process in which the test is given does make a difference in terms of the consistency of the responses. In the early stages of the learning process, before the material has been fully digested, students are still switching strategies as they are testing their algorithms and modifying them whenever it becomes necessary. The presentation of the next question in the test sometimes provides the student with feedback as to the feasibility of his/her algorithm. It was noted that in such cases there is a clear violation of the local independence assumption underlying the Latent Trait models. Since local independence is a critical assumption in the ICC Theory, when testing in early stages of the learning process in which we can't assume a stable latent trait (but rather latent states which are likely to change during the test), applying the ICC models would be inappropriate. In the more advanced stages of learning, when the algorithms have been stabilized, the local independence assumption is satisfied but one is still faced with the fact that the data is multidimensional, due to the existence of different types of algorithms. Thus, again practical Latent Trait models that assume unidimensionality are inappropriate. (Note that while unidimensionality is a sufficient condition for local independence, assuming normality, local independence is only a necessary but not a sufficient condition for unidimensionality). Thus, the development of practical multidimensional Latent Trait models becomes a necessity for this kind of achievement data.

Footnotes

1) For the summary of the verbal responses concerning the subtraction problems, see Appendix 4. Support for this comes from protocols of clinical interviews with students solving the same version of signed-number tests. Those protocols were kindly provided to us by Mr. Seth Chaiklin of the Learning & Development Center at the University of Pittsburgh, to whom we are greatly indebted.

2) Note how many correct answer this wrong algorithm yields.

3) Taken from: Summary of a protocal on the Postman Stories, Feb. 2, 1980. Interviewer: Seth Chaiklin. Summarizers: Nancy Strandmark and Steve Wise.

4) Taken from: Summary of a protocal on the Postman Stories Feb. 1, 1980. Interviewer and Summarizer: Seth Chaiklin.

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Appendices

1. The Fifty-two Item Test in Addition and Subtraction of Signed-Numbers

2. Inconsistent vs. Consistent Responses to Each Task (Percentages) in the January datasets

3. A Typology of Alternative Algorithms used by Students for Solving Signed-Number Problems (January data)

4. Summary of a Student's Verbal Responses Concerning the Subtraction Problems, given in an Interview.

Appendix 1

The Singed-Number Test Used in the January Experiment

Test Items							
I		II		III		IV	
1.	-6-(-8)=2	17.	-1-(-10)=9	33.	-3-(-5)=2	49.	-2-(-11)=9
2.	-7-9=-16	18.	-2-11=-13	34.	-4-6=-10	50.	-5-14=-19
3.	12+-3=?	19.	7+-5=2	35.	15+-6=9	51.	4+-2=2
4.	1-(-10)=11	20.	3-(-12)=15	36.	5-(-7)=12	52.	6-(-8)=14
5.	-3+12=9	21.	-1+10=9	37.	-4+13=9	53.	-2+11=9
7.	8-6=2	23.	7-5=2	39.	4-2=2	55.	9-7=2
8.	-16-(-7)=-9	24.	-12-(-10)=-2	40.	-11-(-2)=-9	56.	-7-(-5)=-2
10.	-14+-5=-19	26.	-10+-1=-11	42.	-7+-5=-12	58.	-10+-8=-18
11.	3+-5=-2	27.	2+-11=-9	43.	6+-8=-2	59.	1+-10=-9
12.	13-(-4)=17	28.	0-(-9)=9	44.	6-(-4)=10	60.	0-(-2)=2
13.	-3-+12=-15	29.	-2-+11=-13	45.	-7-+9=-16	61.	-4-+6=-10
15.	-6+4=-2	31.	-5+3=-2	47.	-4+2=-2	63.	-8+6=-2
16.	2-11=-9	32.	5-14=-9	48.	7-16=-9	64.	4-13=-9

Appendix 2

Inconsistent vs. Consistent Responses to
Each Task (Percentages) in the January Dataset.

	TASK	G1T1					G1T2					G2T1					G2T2				
			Consistent					Consistent					Consistent					Consistent			
		0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4
4	S-(-L)	23	5	56	8	8	31	15	39	5	10	26	2	62	2	7	26	14	43	7	10
2	-S-L	41	10	15	33	0	36	12	10	31	0	24	32	27	38	0	29	21	24	24	2
13	-S-+L	21	0	54	18	8	39	15	33	13	0	42	7	33	14	5	45	14	29	5	7
1	-S-(-L)	23	21	42	0	36	41	18	13	0	28	14	10	29	0	48	71	21	21	2	33
8	-L-(-S)	23	44	23	0	10	15	69	15	0	0	17	64	17	0	2	17	60	17	0	7
7	L-S	21	56	5	0	18	15	54	5	0	0	12	45	0	5	38	17	60	2	0	21
16	S-L	18	56	5	3	18	23	54	8	0	15	36	45	0	5	14	19	45	10	2	24
10	-L+-S	28	46	13	13	0	31	41	18	8	3	43	26	26	2	2	36	24	33	7	0
5	-S+L	21	59	3	8	10	39	46	5	5	5	33	43	7	7	10	41	29	14	5	12
15	-L+S	23	49	15	8	5	21	53	18	5	5	24	43	14	17	2	31	43	19	5	2
3	L+-S	18	67	0	5	10	26	54	15	5	0	26	50	7	5	12	31	38	12	7	12
11	S+-L	21	56	5	8	10	18	41	21	8	13	33	41	12	10	5	29	45	10	12	5

Appendix 3

A Typology of Alternative Algorithms Used by Students for Solving Signed Number Problems (January Data).

Number Line - test1 (T1)															
Algorithm No.:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
n:	6	5	1	3	3	1	1	3	2	2	1	3	2	1	
4. S-(-L)	2	2	2	2	2	2	2	2	2	2	2	4	2	3	
2. -S-L	3	3	3	3	2	2	2	1	2	1	2	1	3	2	
13. -S-+L	3	4	2	2	2	2	2	1	2	2	2	1	3	2	
1. -S-(-L)	4	4	4	4	2	2	2	4	4	2	1	4	4	1	
8. -L-(-S)	1	1	1	1	2	2	1	1	1	2	1	2	1	4	
7. L-S	4	4	1	4	1	1	2	3	1	1	1	4	1	4	
16. S-L	1	1	1	1	4	1	4	3	4	1	1	1	4	4	
Algorithm No.		A	E		A	C			A			B	D		
n:		1	1		1	1			1			1	1		
10. -L+-S	2	4	2	2	1	1	2	3	2		2	1	2	3	2	1
5. -S+L	4	3	1	1	1	1	1	3	1		1	2	4	1	4	2
15. -L+S	1	3	1	4	1	1	1	3	1		1	2	1	4	1	2
3. L+-S	4	2	1	4	1	1	1	1	1		1	3	4	1	3	2
11. S+-L	1	2	1	1	1	1	1	4	1		1	3	1	4	3	2

Postman test1 (T1)																									
	1	2	3	4	5	6	7	8	9	10	11	12	13												
	3	1	1	2	2	1	1	1	1	1	2	1	1												
	2	2	2	2	2	2	2	2	2	2	2	4	3												
	3	2	2	3	3	1	2	1	2	3	3	1	2												
	3	2	2	4	2	2	2	2	2	2	3	3	4	2											
	4	4	2	4	4	2	2	2	2	2	4	4	2	1											
	1	1	2	1	1	2	2	2	2	4	1	1	4												
	4	1	2	4	4	2	1	1	1	1	4	1	1												
	1	4	2	1	1	2	4	1	1	1	1	3	4												
	A	E												C	B	D	F	H							
	1	1												1	1	1	1	1							
	2	2	1			1	1	1	1	3		2	3	3	2	1	1								
	4	1	1			1	1	1	1	4		3	1	3	3	2	2								
	1	1	1			1	1	1	1	4		1	4	3	3	3	2								
	4	1	1			1	1	1	1	1		1	1	1	4	3	3								
	1	1	1			1	1	1	1	1		3	4	4	1	3	3								

Number Line - test2 (T2)															
Algorithm No.:	1	2	3	4	5	6	7	8	9	10	11	12	13		
n:	2	1	2	1	1	2	1	1	1	1	1	1	1		
4. S-(-L)	2	4	2	2	2	2	4	1	2	4	3	4	3		
2. -S-L	3	1	3	1	3	2	1	2	2	1	2	4	2		
13. -S-+L	3	1	2	3	1	2	1	2	2	2	2	4	4		
1. -S-(-L)	4	3	4	2	4	2	2	2	4	2	1	3	1		
8. -L-(-S)	1	1	1	2	1	1	2	2	1	2	4	1	4		
7. L-S	4	1	4	1	4	1	2	1	1	1	1	4	1		
16. S-L	1	3	1	1	1	4	2	1	4	4	4	2	4		
Algorithms:	D			F		C			C		A	B		E	G
n:	2			1		1			1		1	1		1	1
10. -L+-S	2	2	2	2		2	1		2	1	3	2	3	1	2
5. -S+L	4	2	1	1		1	2		1	2	1	3	3	2	1
15. -L+S	1	2	4	1		1	2		1	2	4	1	4	2	1
3. L+-S	4	3	4	2		1	3		1	3	1	4	2	2	4
11. S+-L	1	3	1	2		1	3		1	3	4	2	4	1	1

Postman - test2 (T2)																									
	1	2	3	5	6	7	8																		
	1	5	3	1	1	1	1																		
	1	2	2	2	2	4	2																		
	3	3	3	2	3	1	3																		
	3	2	3	2	2	1	2																		
	4	4	4	4	4	2	4																		
	1	1	1	1	1	2	1																		
	2	4	4	1	1	2	1																		
	2	1	1	4	4	2	1																		
	A				D	H		B	C	F	G	I													
	2				2	2		1	1	1	3	1													
	2				2	1		1	4	2	3	1													
	4				1	2		2	3	2	1	1													
	1				1	2		2	3	1	4	1													
	2				1	3		2	2	1	1	2													
	2				1	3		2	2	2	4	2													

Appendix 4

Summary of a student's Verbal Responses Concerning the Subtraction Problems, Given in an Interview.

-S-(-L)

1 -6-(-8)=-14

Because they're both negatives and so you just add them together.

17 -1-(-10)=

negative, no wait a minute, ... positive 11 'cause you change the signs ... cause they're all the same ... because they're all negatives.

I: so you change all of the signs? S: I think so, I don't really know.

33 -3-(-5)=8

'cause you change all of the signs.

49 -2-(-11)=13

'cause you have to change all of the signs 'cause they're all negatives.

-S-L

2 -7-9=2

oh, I was wrong, sorry (later: because the nine was positive and it was larger)

18 -2-11=9

You subtract 2 from 11 'cause 11 is larger.

34 -4-6=+10

I guess because there are 3 negative signs.

50 -5-14=19

'cause you're changing the signs 'cause they're the same.

S-(-L)

4 1-(-10)=-9

'cause you add one, no you subtract one from the 10, 'cause 10 is the larger number it is negative.

20 3-(-12) =-9

'cause 12-3=9 and 12 is larger so you go by negatives.

36 5-(-7)=12

'cause there is 2 negative signs.

52 6-(-8)

S: -2. I: ok. Tell me why. S: No, not -2, its +14. I: why did you think -2 at first? S: 'cause I was just, I didn't keep track of if there were 2 right next to each other. Because 2 right next to each other canges the signs.

L-S

7 8-6=14

They're both positive so you just add them regularly.

23 7-5=2

'Cause its a regular ... two positive numbers.

39 4-2=2

'Cause they're both the same so you add regularly two

positive number.

55 9-7=2

'Cause they're both the same, positive.

-L-(-S)

8 -16-(-7)=-23

Since they're both the same process you just add regularly equals. I: what's both the same? S: You're just adding...they have the same signs so you add them regularly.

24 -12-(-10)=22

I am going to say +22 because I'm going to change the signs around. I: OK, and why is that? S: because they're all the same sign.

40 -11-(-2)=13

Because there are 3 negative signs so you put them over to the positive.

56 -7-(-5)=12

'Cause they are all the same sign, all negatives.

L-(-S)

12 13-(-4)=11

because the 13 is positive and has the larger number than the negative -- go to positives.

28 0-(-9)=-9

-9 because 0 is just, it just means that...that's -- now

let's see. I: So how did you get -9? S: Oh, no its +9 I: oh, why? S: because there are 2 negative signs next to each other

44 6-(-4)=10

44 because you change the signs 'cause they're right next to each other.

60 0-(-2)=2

'cause there's 2 negative signs right next to each other.

-S-+L

13 -3-+12=15

let's see... I changed it into positive 3 minus positive 12. No,...Oh, positive 3 plus positive 12. I: So positive 15. S: Yeah, I don't know if that's right.

29 -2-+11=13

I: Tell me why. S: 'cause you change the negative signs into positive signs and then you just add regularly. I: Why do you change the negative into positive signs? S: 'cause well, let's see, 'cause there is 2 negative signs right next to each other.

45 -7-+9=16

'cause you change the 2 negative into positive signs.

61 -4-+6=10

'cause there's 2 negative signs right next to each other.

S-L

16 2-11=13

´cause it´s just a regular problem. I: What´s a regular problem?

S: addition of two positive numbers.

32 5-14=-9

´cause you can´t subtract 14 from 5 so you go into negatives.

48 7-16=-9

´cause you cant subtract 16 from 7 so you have to go into

the negatives.

64 4-13=-9

´cause you can´t subtract 13 from 4.

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